Part I Questions

1. If $f^{-1}(x)=\frac{1}{3}x+7$ then which of the following is the correct formula for $f(x)$?

$x=\frac{1}{3}y+7$

$x-7=\frac{1}{3}y$

$3(x-7)=y$

(1) $f(x)=-\frac{1}{3}x-7$ (3) $f(x)=3x-7$

(2) $f(x)=3x-21$ (4) $f(x)=-3x+21$

2. If the linear function $y=-2x+10$ has a domain given by $[-1,4]$ then which of the following is its range?

$y=-2(4)+10=2$

$y=-2(-1)+10=12$

(1) $[2,12]$ (3) $[-5,7]$

(2) $[0,16]$ (4) $[-2,10]$

3. If the expression $\frac{1}{\sqrt{x}}$ was written as x^n then which of the following would be the value of n?

$\frac{1}{\sqrt{x}}=\frac{1}{x^{1/2}}=x^{-1/2}$

(1) $\frac{1}{2}$ (3) $-\frac{1}{2}$

(2) 2 (4) −2

4) Which of the following is *not* a factor of $3x^3+2x^2-12x-8$?

(1) $x-2$ (3) $x+2$

(2) $x-4$ (4) $3x+2$

$x^2(3x+2)-4(3x+2)$

$(3x+2)(x^2-4)$

$(3x+2)(x+2)(x-2)$

5) The quadratic function $f(x)$ has a turning point at $(-3,8)$. If the function g is defined by $g(x)=-f(x+5)$ for all values of x then at which of the following points will $g(x)$ have a turning point?

(1) $(-8,-8)$ (3) $(8,-8)$

(2) $(5,-3)$ (4) $(-2,8)$

flip over x-axis (negate y) then move left (subt. 5 from x)

$(-3,8) \to (-3,-8) \to (-8,-8)$

$f(-3) = -7$

6) The function $f(x)$ is an odd function with $f(3)=7$ and $f(9)=11$. What is the average rate of change of $f(x)$ over the interval $-3 \le x \le 9$?

(1) $\frac{1}{3}$ (3) 3

(2) $\frac{3}{4}$ (4) $\frac{3}{2}$

$f(-x) = -f(x)$

$\frac{f(9)-f(-3)}{9-(-3)} = \frac{11-(-7)}{12} = \frac{18}{12}$

7) The expression $\frac{\sqrt{x}}{\sqrt[6]{x}}$, for $x>0$, is equivalent to which of the following?

(1) $\sqrt[5]{x}$ (3) $\sqrt[6]{x^4}$

(2) $\sqrt[3]{x}$ (4) $\sqrt[3]{x^2}$

$\frac{x^{1/2}}{x^{1/6}} = x^{1/2-1/6} = x^{1/3}$ (*)

$\sqrt[3]{x^1}$

$\sqrt[a]{x^b} = x^{\frac{b}{a}}$

8) If $ab^{\frac{x}{c}} = d$, where a, b, c, and d are all positive constants, then which of the following is the solution for x in terms of a, b, c, and d?

(1) $x=\frac{(ad)^b}{c}$ (3) $x=c\log_b(ad)$

(2) $x=\frac{(d/a)^c}{b}$ (4) $x=c\log_b\left(\frac{d}{a}\right)$

$b^{x/c} = \frac{d}{a}$

$\frac{x}{c} = \log_b\left(\frac{d}{a}\right)$

$x = c\cdot\log_b\left(\frac{d}{a}\right)$

$b^x = \left(\frac{d}{a}\right)^c$

$x = \log_b\left(\frac{d}{a}\right)^c$

$x = c\log_b\frac{d}{a}$

$5\cdot 3^{\frac{x}{2}} = 7$

$3^{\frac{x}{2}} = \frac{7}{5}$

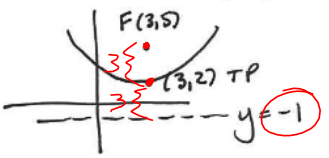
$\frac{x}{2} = \log_3\frac{7}{5}$

$x = 2\log_3\frac{7}{5}$

9) If a parabola has a focus of $(3,5)$ and a turning point of $(3,2)$ then which of the following represents the equation of its directrix?

(1) $x=3$ (3) $x=5$

(2) $y=-1$ (4) $y=8$



2 − 3 = −1

10) For the exponential function $f(x)=a(b)^x$ we know that $f(3)=17$ and $f(7)=3156$. Which of the following is closest to the value of b?

(1) 1.87 (3) 3.69

(2) 2.91 (4) 4.35

$y=ab^x$

$\frac{3156}{17}=\frac{a\cdot b^7}{ab^3}$

$\frac{3156}{17}=b^4$ $b=3.691$

11) A line is drawn from the vertex of the parabola $y=(x+5)^2+9$ to the center of the circle whose equation is $(x-4)^2+(y-6)^2=100$. Which of the following is the slope of the line?

(1) $\frac{5}{2}$ (3) 3

(2) $-\frac{1}{3}$ (4) $-\frac{5}{3}$

Vertex: $(-5,9)$

Center: $(4,6)$

$\text{Slope}=\frac{9-6}{-5-4}=\frac{3}{-9}$

12) If $g(x)=2x-1$ and $f(x)=x^2+2x$ then $f(g(x))=$ $(2x-1)^2+2(2x-1)$

(1) x^2+4x+1 (3) $2x^3+3x^2-2x$

(2) $2x^2+6x+3$ (4) $4x^2-1$

$4x^2-4x+1+4x-2$

13) If the function $f(x)$ has zeroes at -4 and 10, then which of the following would have zeroes at -2 and 5?

$\therefore f(-4)=0$

$f(10)=0$

(1) $f(x+2)$ (3) $\frac{1}{2}f(x)$

(2) $f(2x)$ (4) $f\left(\frac{1}{2}x\right)$

FREE RESPONSE QUESTIONS: Clearly indicate the necessary steps and explain your reasoning for each of the following problems.

14) Solve the following equation algebraically.

$\frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

$3^{x^2} = \left(\frac{1}{9}\right)^{x-4}$

$3^{x^2} = (3^{-2})^{x-4}$

$x^2 = -2x + 8$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4$ or $x = 2$

15) Solve the following system of equations algebraically.

$2x + y - z = 19$

$-3x + 2y + 4z = -9$

$5x - 2y + 2z = 7$

$4① + ② \rightarrow 5x + 6y = 67$

$2① + ③ \rightarrow 9x = 45$

$8x + 4y - 4z = 76$ ← 4

$\oplus\ -3x + 2y + 4z = -9$ ←

$5x + 6y = 67$

$4x + 2y - 2z = 38$

$\oplus\ 5x - 2y + 2z = 7$

$9x = 45$

$x = 5$

$x = 5$

$5(5) + 6y = 67$

$6y = 42$

$y = 7$

$2(5) + 7 - z = 19$

$-z = 2$

$z = -2$

$(5, 7, -2)$

$\log_2(0-4) = \log_2(-4)$

16) Consider the function $f(x)=\log_2(x-4)$.

(a) Explain why $x=0$ is *not* part of the domain of $f(x)$.

$f(0)=\log_2(-4)$ impossible

you cannot raise 2 to any power and get −4.

(b) Max proposed the function $g(x)=2^x+4$ as the inverse of $f(x)$. Supply numerical evidence that either supports or disproves Max's claim.

$g(1)=6$ $f(6)=\log_2 2=1$ ✓

$g(0)=5$ $f(5)=\log_2 \phi=0$ ✓

$g(2)=8$ $f(8)=\log_2 4=2$ ✓

$g(x)$ is the inverse of $f(x)$.

17) Solve the following equation algebraically.

$(2x+1)^2-3(2x+1)-10=0$

Let $A=2x+1$

$A^2-3A-10=0$

$(A-5)(A+2)=0$

$A=5 \quad A=-2$

$2x+1=5 \quad 2x+1=-2$

$x=2, \quad x=-\frac{3}{2}$

$4x^2+4x+1-6x-3-10=0$

$4x^2-2x-12=0$

$y = 400(1.56)^x$

18) The population of bacteria an experiment can be modeled by $p(d) = 400(1.56)^d$, where d is the number of days the population has been growing.

(a) Find the average rate the population has been growing over the interval $0 \le d \le 7$. Round to the nearest integer and include appropriate units.

0, 400

7, 8993.57

$\frac{d(7)-d(0)}{7-0} = 1227.653$

$\frac{8993.57-400}{7-0}$ | 1228 bacteria per day

$y = 400(1.56)^0$

$y = 400$

$y = 400(1.56)^7$

$y = 8993.57$

(b) If the population of bacteria was modeled as a function of the number of hours, h, such that $p(h) = 400b^h$, then what would be the value of b to the nearest thousandth? Show or explain how you found your answer.

Take daily base (1.56) and find the hourly

$(1.56)^{\frac{1}{24}} = 1.019$

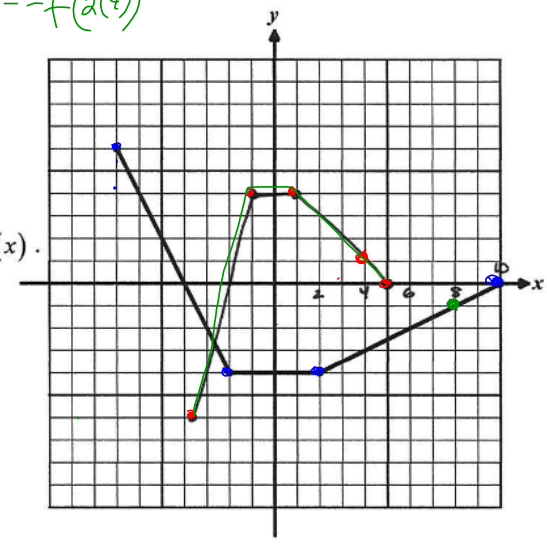
19) Given the function $f(x)$ shown graphed and $g(x) = -f(2x)$ answer the following questions.

$g(4) = -f(2(4))$

(a) Evaluate $g(4)$. Show how you arrived at your answer.

$g(4) = -f(8) = -(-1)$

$= +1$



(b) Produce a graph of $g(x)$ on the same grid as $f(x)$.

(-7, 6) .. $(-\frac{7}{2}, -6)$

(-2, -4) (-1, 4)

(2, -4) (1, 4)

(10, 0) (5, 0)

(c) State the domain and range of $g(x)$.

$-3.5 \le x \le 5$ $-6 \le y \le 4$